(8)

[This question paper contains 4 printed pages]

**Your Roll No.** : 2019

**Sl. No. of Q. Paper** : **7461** **J**

Unique Paper Code : 32351101 - OC

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Calculus

Semester : I

***Time : 3 Hours*** ***Maximum Marks : 75***



**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** the sections are compulsory.

(c) All questions carry equal marks.

(d) Use of non-programmable scientific calculator is allowed.

## Section-I

**Note :** Attempt any **four** questions from this **Section.**

**1.** If $y = \sin^{-1} x$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n = 0$.

**2.** Sketch the graph of $f(x) = \frac{1}{x+1} + \frac{1}{x-1}$ by finding intervals of increase and decrease, critical points, points of relative maxima and minima, concavity of the graph and inflection points.

P.T.O.

**3.** Evaluate analytically following problem :

$$\lim_{x\to\infty}\left(x\sin^{-1}\left(\frac{1}{x}\right)\right)^{x}$$

**4.** Suppose a manufacturer estimates that, when the market price of a certain product is p, the number of units sold will be $= -6\ln\left(\frac{p}{40}\right)$. It is also estimated that the cost of producing these *x* units will be $C(x) = 4xe^{-\frac{x}{6}} + 30$.

(a) Find the average cost, the marginal cost, and the marginal revenue for this production process.

(b) What level of production *x* corresponds to maximum profit ?

**5.** Sketch the graph of the curve in polar coordinates $r = 4 - 4\cos\theta$.

## Section - II

**Note :** Attempt any **four** questions from this **Section.**

**6.** Find the reduction formula for $\int x^n e^x dx$ and hence evaluate $= \int_0^1 xe^{-\sqrt{x}} dx$.

**7.** Find the volume of the solid generated when the region enclosed by the curves $y = \sqrt{25 - x^2}$ and y = 3, is revolved about x-axis.

**8.** Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curve $y = \frac{1}{x^3}, x = 1, x = 2, y = 0$ is revolved about the line $x = -1$.

**9.** Find the exact arc length of the curve $y = \frac{x^6 + 8}{16x^2}$ from $x = 2$ to $x = 3$.

**10.** Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2}$, $-2 \leq y \leq 2$. about y-axis.

## Section- III

**Note :** Attempt any **three** questions from this **Section**.

**11.** State the reflection properties of the conic sections : parabolas, ellipses and hyperbolas with diagram.

**12.** Find an equation for the parabola that has its vertex at (1,2) and its focus at (4,2).

**13.** Describe the graph of the equation $9x^2 + 4y^2 + 18x - 24y + 9 = 0$ with rough sketch label the foci, vertices and the ends of minor axis.

**14.** Trace the conic $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$ by rotating the coordinate axes to remove the xy term.

## Section - IV

**Note :** Attempt any **four** questions from this **Section.**

**15.** Find the position vector and velocity vector if acceleration vector with initial conditions are given as $A(t) = (\cos t)\hat{i} - (t \sin t)\hat{k}$; $R(0) = \hat{i} - 2\hat{j} + \hat{k}$;

$V(0) = 2\hat{i} + \hat{k}$.

**16.** A boy standing at the edge of a cliff throws a ball upward at a $30^{\circ}$ angle with an intial speed of 64 ft/s. Suppose that when the ball leaves the boy's hand, it is 48 ft above the ground at the base of the cliff.

(a) What are the time of flight of the ball and its range ?

(b) What are the velocity of the ball and its speed at impact ?

(c) What is the highest point reached by the ball during its flight ?

**17.** Find the tangential and normal components of acceleration of an object that moves with position vector $R(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + (\sin t)\hat{k}$.

**18.** An object moves along the curve in the plane described in polar form $r = 3 + 2\sin t$; $\theta = t^3$.

Find its velocity and acceleration in terms of unit polars $U_r$ and $U_\theta$.

**19.** Find the curvature and radius of curvature at the stated point for a curve

$x = e^t \cos t, \quad y = e^t \sin t, \quad z = et \quad t = 0$

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**Your Roll No.** (9) : 2019

**Sl. No. of Q. Paper** : **7462** **J**

Unique Paper Code : 32351102 - OC

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Algebra

Semester : I

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Attempt any **two** parts from each questions.

(iii) **All** questions are compulsory.

**1.** (a) Find the polar representation for the complex number 6

$z = 1 - \text{Cos}a + i\,\text{Sin}a, \quad a \in [0, 2\pi)$

(b) Solve the equation $(2 - 3i)z^6 + 1 + 5i = 0$. 6

(c) Compute $z^n + \frac{1}{z^n}$, if $z + \frac{1}{z} = \sqrt{3}$. 6



P.T.O.

**2.** (a) Define ~ on $\mathbb{Z}$ by a~b if and only if $2a + 3b = 5n$ for some integer n. Prove that ~ defines an equivalence relation on $\mathbb{Z}$. 6

(b) Define $f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 3x^3 - x$.

(i) Is $f$ one-to-one ?

(ii) Is $f$ onto ?

Justify each answer. 6

(c) Show that the open intervals (0, 1) and (1, 2) have the same cardinality. 6

**3.** (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that $17369x + 5472y = 1$. 6

(b) (i) Show that $3^6 \equiv 1 \pmod 7$ and hence evaluate $3^{60} \pmod 7$.

(ii) Find all integers $x \pmod{12}$ that satisfy $9x \equiv 3 \pmod{12}$. 6

(c) Use the Principle of Mathematical Induction to prove $2^{2n} - 1$ is divisible by 3, $\forall n \geq 1$. 6

**4.** (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_2 - 6x_3 = -3$$

(b) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$. Show that the equation $Ax = b$ may not be consistent for every $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Also describe the set of all vectors b for which $Ax = b$ is consistent. 6.5

(c) Determine h and k such that the solution set of the given system 6.5

$x_1 + 3x_2 = k$

$4x_1 + h\,x_2 = 8$

(i) is empty.

(ii) contains a unique solution.

(iii) contains infinitely many solutions.

5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is $B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$.

Balance the chemical equation using the vector equation approach. 6.5

(b) Find the value of h for which the following vectors are linearly dependent. Also find a linear dependence relation among them. 6.5

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$$

(c) A linear transformation $T : R^2 \to R^2$ first performs a vertical shear that maps $e_1$ into $e_2 - 2e_1$, leaves the vector $e_2$ unchanged and then reflects point through the line $x_2 = x_1$

(i) Find Matrix A such that $T(x) = Ax$, $x \in R^2$.

(ii) Find x such that $T(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. 6.5

**6.** (a) Given :

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

(i) Show that the matrix A is row equivalent to $I_3$.

(ii) Find inverse of A and hence find inverse of $A^T$. 6.5

(b) Find a basis for column space for the matrix A 6.5

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$$

(c) Is $\lambda = 4$ an eigen value of the matrix A ?

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

If so, find eigen space of A corresponding to eigen value $\lambda = 4$. 6.5

This question paper contains 4 printed pages]

**Roll No.** 2019

10

S. No. of Question Paper : **8597**

Unique Paper Code : **32351101**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**



Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*All* sections are compulsory.

*All* questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

## Section I

Attempt any *four* questions from Section I.

1. State Leibnitz's theorem for finding $n$th derivative of product of two functions. If $y = a\cos(\ln x) + b\sin(\ln x)$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

P.T.O.

2. Evaluate the following limit :

$$\lim_{x \to 0^+} x^{\sin x}$$

3. Find the intervals of increase and decrease of the following function, discuss its concavity and then sketch its graph $y = (x+1)^2(x-5)$.

4. Sketch the graph of the polar curve $r = 3\cos 2\theta$.

5. A manufacturer estimates that when '$x$' units of a particular commodity are produced each month, the total cost (in dollars) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to the maximum profit.

**Section II**

Attempt any *four* questions from Section II.

6. Find a reduction formula for $\int \operatorname{cosec}^n x\, dx$, $n \geq 2$ is an integer. Evaluate $\int \operatorname{cosec}^4 x\, dx$.

7. Find the volume of the solid generated when the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the $x$-axis.

8. The base of a certain solid is enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross-section perpendicular to the $x$-axis is a semicircle with its diameter across the base. Find the volume of the solid.

9. Find the arc length of the parametric curve :

$$x = (1 + t)^2,\ y = (1 + t)^3,\ 0 \leq t \leq 1.$$

10. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the $x$-axis.

## Section III

Attempt any *three* questions from Section III.

11. Find the equation of the parabola whose focus is $(-1, 4)$ and directrix is $x = 5$.

12. Find the equation of the hyperbola whose foci are $(1, 8)$ and $(1, -12)$ and vertices are 4 units apart.

13. Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

14. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

## Section IV

Attempt any *four* questions from Section IV.

15. Evaluate :

$$\lim_{t \to 0^+} [\frac{\sin 3t}{\sin 2t} \hat{i} + \frac{\log(\sin t)}{\log(\tan t)} \hat{j} + (t \log t)\hat{k}].$$

16. The acceleration of a moving particle is $\vec{A}(t) = 24t^2\hat{i} + 4\hat{j}$. Find the particle's position as a function of $t$ if $\vec{R}(0) = \hat{i} + 2\hat{j}$ and $\vec{v}(0) = 0$.

17. If a shot putter throws a shot from a height of 5 ft with an angle of 46° and initial speed of 25 ft/sec, what is the horizontal distance of the throw ?

18. Find $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$ at $t = \frac{\pi}{4}$.

19. Show that the curvature of the polar curve $r = e^{\alpha\theta}$ is inversely proportional to $r$.

This question paper contains **4** printed pages]

**Roll No.** | | | | | | | |2|0|1|9|

(11)

S. No. of Question Paper : **8617**

Unique Paper Code : **32351102**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**



Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

All *six* questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. 5

   (*b*) Find all the rational roots of $96y^3 - 16y^2 - 6y + 1 = 0$. 5

   (*c*) (*i*) Find the geometric image of the complex numbers z, such that $|z + i| \geq 2$.

   (*ii*) Find the polar representation of the complex number $z = -4i$ and find $Arg\ z$. 2,3

2. (*a*) Find all complex numbers $z$ such that $|z| = 1$ and

$$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1.$$ 5

P.T.O.

(*b*) Solve the equation : 5

$$z^4 = 5(z-1)(z^2 - z + 1).$$

(*c*) Show that : 5

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \cdot$$

3. (*a*) For $(x, y)$ and $(u, v)$ in $R^2$, define $(x, y) \sim (u, v)$ if $x^2 + y^2 = u^2 + v^2$.

Prove that $\sim$ defines an equivalence relation on $R^2$. Find equivalence classes of (1, 0) and (1, 1). 6

(*b*) Suppose $f : A \to B$ and $g : B \to C$ are functions :

(*i*) If $gof$ is one-to-one and $f$ is onto, prove that $g$ is one-to-one.

(*ii*) If $gof$ is onto and $g$ is one-to-one, prove that $f$ is onto. 3,3

(c) Prove that the intervals $(0, 1)$ and $(0, \infty)$ have the same cardinality. 6

4. (*a*) (*i*) Suppose $a$ and $b$ are integers and $p$ is a prime such that $p|ab$. Then prove that $p|a$ or $p|b$.

(*ii*) Find the quotient $q$ and the remainder $r$ as defined in division algorithm. If $a = -517$ and $b = 35$. 3½,3 (*b*) Using Euclid's Algorithm, find integers $x$, $y$ such that $150x + 284y = 4$. 6½

(*c*) Using Principle of Mathematical Induction prove that for any $x \in \mathrm{R}$, $x > -1$, $(1+x)^n \geq 1+nx$, $\forall n \in \mathrm{N}$. 6½

5. (*a*) Consider the following system of linear equations :

$$x_1 + 3x_2 + x_3 = 1$$
$$-4x_1 - 9x_2 + 2x_3 = -1$$
$$-3x_2 - 6x_3 = -3$$



Write the matrix equation and the vector equation of the above system of equations. Find the general solution in parametric vector form by reducing the augmented matrix to echelon form. 7½

(*b*) Let $\mathrm{T} : \mathrm{R}^2 \to \mathrm{R}^3$ be a linear transformation such that :

$$\mathrm{T}(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$$

(*i*) Find standard matrix of T.

(*ii*) Is T one-to-one ? Is T onto ? Justify your answers.

(*iii*) Find X such that T(X) = (–1, 4, 9). 7½

(*c*) (*i*) Let $\mathrm{A} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ Find an eigenvector corresponding to an eigenvalue $\lambda = 3$.

(*ii*) Show that if $\lambda$ is an eigenvalue of A and $p(t) = c_0 + c_1 t + c_2 t^2 + ..... + c_n t^n$, then one eigenvalue of $p(\text{A})$ is $p(\lambda)$. 5,2½

6. (*a*) (*i*) Using homogeneous coordinates, find the $3 \times 3$ matrix that produce the following composite transformation : Reflect points through the *x*-axis, and then rotate 30° about the origin.

(*ii*) Show that $\text{H} = \{(a, b, c) \in \text{R}^3 \mid b = 2a + 3c\}$ is a subspace of $\text{R}^3$. 5,2½

(*b*) Let $\text{S} = \{v_1, v_2, v_3, v_4\}$ where $v_1 = (1, 2, 2)$, $v_2 = (3, 2, 1)$, $v_3 = (11, 10, 7)$, $v_4 = (7, 6, 4)$. Find a basis for the subspace W = span S of $\text{R}^3$. What is dim W ? 7½

(*c*) Compute the rank and nullity of the matrix A. Show that rank A + nullity A = number of columns of A.

$$\text{A} = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -1 & 3 \\ 7 & -8 & 3 \\ 5 & -7 & 0 \end{bmatrix}.$$ 7½

[This question paper contains 3 printed pages.]

Sr. No. of Question Paper : 8895 (12) Your Roll No. 2019

Unique Paper Code : 235101

Name of the Course : **B.Sc. (Hons.) Mathematics**

Name of the Paper : MAHT 101 Calculus-I

Semester : I

Duration: 3 Hours Maximum Marks: 75

**Instruction for Candidates**

1) Write your Roll No. on the top immediately on receipt of this question paper.
2) All the sections are compulsory.
3) All questions carry equal marks.
4) Use of non-programmable Scientific Calculators is allowed.



## Section I

*Attempt any* **four** *questions from* **Section I**.

1. If $y = (1 - x^2)^{-\frac{1}{2}} Sin^{-1}x$, when $-1 < x < 1$ and $-\frac{\pi}{2} < Sin^{-1}x < \frac{\pi}{2}$, then show that

$$(1 - x^2)y_{n+1} - (2n + 1)xy_n - n^2 y_{n-1} = 0$$

2. Sketch the graph of the function $f(x) = 4 + \frac{2x}{x-3}$ by determining all critical points, interval of increase and decrease, point of relative maxima and minima, concavity of the graph, inflection point and horizontal and vertical asymptotes.

3. Evaluate :

$$\lim_{x \to +\infty} [x - \log(x^3 - 1)]$$

4. Sketch the graph of $r = 5 - 2Cos\theta$ in polar coordinates.

5. When the market price of a certain product is $p$, then number of units sold will be

$$x = -6\log\left(\frac{P}{40}\right)$$

It is also estimated that the cost of producing these x units will be

$$C(x) = 4xe^{\left(-\frac{x}{6}\right)} + 30$$

(a) Find the average cost, the marginal cost, and the marginal revenue for this production process.

(b) What level of production x corresponds to maximum profit?



## Section II

*Attempt any* **four** *questions from* **Section II**.

6. Find the reduction formula for $\int \sec^n x\, dx$ where $n \geq 2$ is an integer. Hence, evaluate $\int \sec^5 x\, dx$ .

7. Find the volume of solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

8. Use cylindrical shell method to find the volume of the solid generated when the region enclosed by the curves $xy = 1, x + y = 5$ is revolved about the $x-$ axis.

9. Find the arc length of the parametric curve $x = a(t + Sin\, t),\ y = a(1 - \cos t)$ for $-\pi \leq t \leq \pi$.

10. Find the area of the surface generated by revolving the parametric curve $x = Cos^2 t, y = Sin^2 t, 0 \leq t \leq \pi/2$ about the y-axis.

## Section III

*Attempt any* **four** *questions from* **Section III**.

11. Find the equation of the hyperbola passing through the origin with asymptotes $y = 2x + 1$ and $y = -2x + 3$ .

12. Find the equation of the ellipse having foci at $(0, \pm 6)$, length of the minor axis 16

13. Identify and sketch the following curve :

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0.$$

14. Identify and sketch the following curve :

$$y^2 - 8x - 6y - 23 = 0.$$



## Section IV

*Attempt any* **four** *questions from* **Section IV**.

15. If $F(t)$ is a differentiable vector valued function of t of constant length then show that $F(t)$ is orthogonal to its derivative for all t.

16. Evaluate $\int_0^{\frac{\pi}{4}} F(t)\,dt$, where $F(t) = (\sec^2 t, -2\cos t, 1)$.

17. Express the acceleration of the particle in the form $a_T T + a_N N$, where T is the unit tangent vector and N is the unit normal vector, given that the particle moves so that its position at any time t is $r(t) = \left(e^t \cos t, e^t \sin t, \sqrt{2}e^t\right), t > 0$.

18. Find the curvature and radius of curvature of the twisted cubic for a curve $\mathbf{r}(t) = (t, t^2, t^3)$ at a general point and at $(0, 0, 0)$.

19. A projectile is fired from ground level at angle 30° with muzzle speed of 80 ft/s. Find time of flight and the range.

Sl No of Q.P: 8896 2019

Unique Paper Code : 235103 (13)

Name of the Paper : Analysis- I

Name of the Course : B.Sc. (Hons) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

*(Write your Roll No. On the top immediately on receipt of this question paper.)*

*All questions are compulsory.*

*Attempt any **three** parts from each question.*



1. (a) State and prove the triangle inequality and show that

   $||a| - |b|| \leq |a - b| \quad \forall a, b \in \mathbf{R}.$

   (b) Let S be a nonempty bounded set in **R**. Let $a > 0$ and $aS = \{as : s \in S\}$.

   Show that $\sup(aS) = a \sup(S)$.

   (c) State and Prove Archimedean property of real numbers.

   (d) If $y > 0$, show that there exists $n \in \boldsymbol{N}$ such that $\frac{1}{2^n} < y$.

2. (a) Show that intersection of an arbitrary family of closed sets is a closed set. Is this result true for an arbitrary family of open sets? Justify your answer.

   (b) Define Limit point of a set of real numbers. Prove that a point $p \in \boldsymbol{R}$ is a limit point of a set S if and only if every neighbourhood of $p$ contains infinitely many points of S.

   (c) Let $(x_n)$ be a sequence of real numbers such that $(x_n)$ converges to $x$, $x > 0$ show that there exists a natural number k such that $\frac{x}{2} < x_n < 2x$, $\forall\, n \geq k$.

   (d) Find the following limits and use the definition of the limit of a sequence to establish the limits

i. $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n+7}}\right)$.

ii. $\lim_{n\to\infty}\left(\frac{2n}{n+2}\right)$.

3. (a) Let $(x_n)$ be sequence of positive real numbers such that $L = \lim_{n\to\infty} \frac{x_{n+1}}{x_n}$ exists. If $L < 1$ then show that $(x_n)$ converges and $\lim_{n\to\infty} x_n = 0$.

(b) State and prove the monotone convergence theorem.

(c ) Establish the convergence or divergence of the sequence $(x_n)$ where

$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}, \forall\ n \in N .$$



(d) Show that the following sequences are divergent

i. $((-1)^n)$

ii. $(\sin\left(\frac{n\pi}{3}\right))$.

4. (a) Show that sequence of real numbers is Cauchy if and only if it is convergent.

(b) If $0 < r < 1$ and $|x_{n+1} - x_n| < r^n$, $\forall\ n \in N$. Show that $(x_n)$ is a Cauchy sequence.

(c) State Root test for series of real numbers and show that the series $\sum \frac{1}{n^n}$ is convergent.

(d) Examine the following series for convergence

i. $\sum \frac{(100)^n}{n!}$

ii. $\sum (\sqrt{n+1} - \sqrt{n})$

5. (a) Suppose $\sum a_n$ is a series, where $a_n \geq 0$ and $|b_n| \leq a_n$. Then prove that $\sum a_n$ is convergent implies $\sum b_n$ is convergent. Is the series $\sum \frac{(\cos(n)+2)}{3^n}$ convergent? Justify your answer.

(b) Using integral test, prove that the series $\sum \frac{1}{n\,(\log n)^p}$ converges if and only if $p > 1$.

(c) Define conditional convergence of an infinite series. Prove that the series $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$ is conditionally convergent.

(d) Show that the series $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ converges absolutely for all values of $x$.

[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 8897 (14) Your Roll No. 2019

Unique Paper Code : 235104

~~Paper Code : MAHT 103~~

Name of the Course : **B.Sc. (Hons.) Mathematics – I**

Name of the Paper : Algebra-I

Semester : I

Duration: 3 Hours Maximum Marks:75



**Instruction for Candidates**

1) All six questions are compulsory.
2) Do any two parts from each question.
3) Marks for each part of a question are written against the question in the margin.

1. a) Find the polar representation of complex number 6

$$z = \cos a - i \sin a \, , a \in [0,2\pi)$$

b) Compute the following 6

$$z^n + \frac{1}{z^n}, \text{ if } z + \frac{1}{z} = \sqrt{3}.$$

c) Find the quadratic equation whose roots are the cubes of the roots of the equation $x^2 - px + q = 0$. 6

2. a) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ if and only if $\frac{a}{b} \in \mathbb{Q}$ 6

i. Prove that $\sim$ defines an equivalence relation .

ii. What is an equivalence class of 1? Show that $\overline{\sqrt{3}} = \overline{\sqrt{12}}$ .

b) Given three consecutive integers $a, a+1, a+2$, prove that one of them is divisible by 3. 6

c) ~~a)~~ Define $f: \mathbb{Z} \to \mathbb{Z}$ by 6

$$f(x) = 3x^3 + x .$$

Determine whether or not $f$ is one to one and/or onto .

3. a) Use mathematical induction to establish that for all $n \geq 1$, $8^n - 3^n$ is divisible by 3. 6

b) Show that the set of rational numbers is countable. 6

c) Find all integers $x, 0 \le x < 6$, satisfying the following congruence $4x \equiv 2 \pmod 6$. 6

4. a) Find the general solution of the system $6\frac{1}{2}$

$$\begin{aligned} 2x_1 - x_2 + x_3 + 2x_4 &= 0 \\ -2x_1 + 4x_2 - x_3 + 2x_4 &= -5 \\ x_1 - 6x_2 + 3x_3 + x_4 &= 7 \\ 4x_1 - 6x_2 + x_3 - 4x_4 &= 9 \end{aligned}$$



by reducing the coefficient matrix to echelon form.

b) Determine whether $\boldsymbol{b}$ belongs to the linear span of $\boldsymbol{a_1}, \boldsymbol{a_2}$ and $\boldsymbol{a_3}$, where $6\frac{1}{2}$

$$\boldsymbol{a_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \boldsymbol{a_2} = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, \boldsymbol{a_3} = \begin{pmatrix} -6 \\ 7 \\ 5 \end{pmatrix}, \text{ and } \boldsymbol{b} = \begin{pmatrix} 11 \\ -5 \\ 9 \end{pmatrix}.$$

c) Balance the following chemical equation $6\frac{1}{2}$

$$NaHCO_3 + H_3C_6H_5O_7 \dashrightarrow Na_3C_6H_5O_7 + H_2O + CO_2.$$

5. a) For what values of $h$ the vectors $\boldsymbol{v_1}, \boldsymbol{v_2}$ and $\boldsymbol{v_3}$ given below $6\frac{1}{2}$

$$\boldsymbol{v_1} = \begin{pmatrix} \mathbf{3} \\ \mathbf{-6} \\ \mathbf{1} \end{pmatrix}, \boldsymbol{v_2} = \begin{pmatrix} \mathbf{-6} \\ \mathbf{4} \\ \mathbf{-3} \end{pmatrix}, \boldsymbol{v_3} = \begin{pmatrix} \mathbf{9} \\ \boldsymbol{h} \\ \mathbf{3} \end{pmatrix},$$

are linearly dependent?

b) Let $\boldsymbol{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \boldsymbol{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \boldsymbol{y_1} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, and $y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be such that $\boldsymbol{Te_1} = \boldsymbol{y_1}$ and $T\boldsymbol{e_2} = \boldsymbol{y_2}$. Find $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. $6\frac{1}{2}$

c) (i) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Show that $T$ is one-to-one if and only if $T(\boldsymbol{x}) = \mathbf{0}$ has only the trivial solution. $6\frac{1}{2}$
(ii) Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x_1, x_2) = (x_1 + x_2, x_2)$ is one-to-one.

6. a) Find the standard matrix of the horizontal sheer transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ that leaves $\boldsymbol{e_1}$ unchanged and maps $\boldsymbol{e_2}$ into $\boldsymbol{e_2} + \boldsymbol{2e_1}$. $6\frac{1}{2}$

b) Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transform and $A$ be standard matrix representation of $T$. Show that $T$ is invertible linear transformation if and only if $A$ is an invertible matrix. $6\frac{1}{2}$

c) Determine the rank of the matrix $6\frac{1}{2}$

$$\begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{pmatrix}.$$

(15)

Roll No. 2019..........

Sr. No of Question Paper : ~~6406~~ 1301

Unique Paper Code : 205162

Name of the Paper : Hindi Lower Qualifying

Name of the Course : B.A. (Hons.) /B.Sc. Hons. Maths

Semester : I

Duration 3 Hours

Maximum Marks- 100

(इस प्रश्न पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए)

प्रश्न–1 हिन्दी भाषा के भौगोलिक विस्तार का परिचय दीजिए।

**अथवा**

भाषा की परिभाषा देते हुए उसका महत्त्व स्पष्ट कीजिए। 10

प्रश्न–2 भाषा और बोली का अंतर स्पष्ट करते हुए उसकी विशेषताएँ बताइए।

**अथवा**

हिन्दी वर्तनी के मानक रूप की समस्याओं पर प्रकाश डालिए। 10

प्रश्न–3 किन्हीं **तीन** वाक्यों में से निर्देशानुसार व्याकरणिक रूप छाँटकर लिखिए : 3

(i) राम पुस्तक पढ़ता है। (संज्ञा)

(ii) वह स्कूल जाता है। (सर्वनाम)

(iii) मुस्कान धीरे–धीरे खाना खाती है। (क्रिया–विशेषण)

(iv) श्याम तेज़ दौड़ता है। (विशेषण)



(ख) किन्हीं **तीन** शब्दों के दो–दो पर्याय लिखिए : 3

आँख, पर्वत, सूर्य, घर, पुष्प, बादल

(ग) किन्हीं **छह** शब्दों के विलोम लिखिए : 3

सुख, नवीन, प्रकाश, आयात, दिन, सपूत, मित्र, हार

(घ) किन्हीं **चार** शब्दों के शुद्ध रूप लिखिए : 2

करम, व्यक्ती, कवी, गृहणी, प्रतीभा, व्यापत

(ङ) किन्हीं **तीन** वाक्यों के शुद्ध रूप लिखिए : 3

(i) मेरे अनेकों मित्र है।

(ii) मुझे एक फूलों की माला चाहिए।

(iii) मेरे को आज शादी में जाना है।

(iv) एक गिलास गर्म गाय का दूध लाओ।

(v) राम क्रिकेट खेलती है।

(च) किन्हीं **तीन** मुहावरों के अर्थ लिखकर वाक्य बनाइए : 3

(i) घी के दिये जलाना

(ii) आँख का तारा

(iii) दाँतों तले उंगली दबाना

(iv) आग-बबूला होना

(v) नौ दो ग्यारह होना।

(छ) किन्हीं **तीन** लोकोक्तियों का वाक्य में प्रयोग कीजिए : 3

(i) ऊँची दुकान फीका पकवान

(ii) नाँच न जाने आँगन टेढ़ा

(iii) चौर की दाढ़ी में तिनका

(iv) बंदर क्या जाने अदरक का स्वाद

(v) चिराग तले अँधेरा।

प्रश्न–4 किसी **एक** का पल्लवन कीजिए (अधिकतम 200 शब्द) 20

(i) भ्रष्टाचार

(ii) समय का सदुपयोग

**अथवा**

किसी **एक** विषय पर अनुच्छेद लिखिए :

(i) दिल्ली मेट्रो

(ii) बिन कम्प्यूटर सब सून

प्रश्न–5 (क) अपने प्रधानाचार्य को फीस माफी हेतु प्रार्थना पत्र लिखिए। 10

**अथवा**

परीक्षा में सफलता हेतु शुभकामना देते हुए मित्र को पत्र लिखिए।

(ख) अपने क्षेत्र में नारी सुरक्षा की समस्या हेतु किसी समाचार पत्र के संपादक को पत्र लिखिए। 10

**अथवा**

समाचार–पत्र के संपादक को अपने मौहल्ले की सफाई की समस्या हेतु पत्र लिखिए।

प्रश्न–6 किसी **एक** विषय पर निबंध लिखिए। : 20

(i) भारत निर्माण में युवाओं की भूमिका

(ii) सोशल मीडिया का प्रभाव

(iii) सिनेमा और साहित्य

(iv) आर्थिक सुधारों में जीएसटी (GST) की भूमिका